Diagonal Matrices

Square the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Q: What happened?

Suppose $k \in \mathbb{N}$. Calculate A^k.

Let

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$$

One can write

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Exercise: Calculate A^{20}.

Zero Divisors

Notice that

$$\begin{pmatrix} 2 & 4 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} -2 & -2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Do the multiplication in the reverse order

Idempotents

Notice that $\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)$ squared is itself. For which matrices $A \in \mathrm{Mat}_{2\times 2}(\mathbb{R})$ does

$$A \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = A?$$

Nilpotents

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$ is the zero matrix.